A Discrete Master Equation for Dispersion-Tuned Fiber Lasers

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Abstract: We developed a discrete master equation describing pulse propagation through the different components of dispersion-tuned fiber lasers, which cannot be achieved using average models. The model is validated through numerical simulations and experimental data.

OCIS codes: 140.3510, 140.3600.

1. Introduction and Model

Dispersion-tuned actively modelock lasers are fully electronically controlled since the pulse duration, wavelength and repetition rates are defined through the modulation. These programmable lasers have been used in numerous applications such as sensing [1], optical coherence tomography (OCT) and coherent anti-stokes Raman spectroscopy (CARS) [2]. An adequate model of these lasers is required in order to improve their design. The main modelling effort has been on average master equations [3], which average all the effects of the optical elements in the cavity. We present here a more detailed modelling based on a discrete master equation which considers each optical element separately and thus allows us to describe the propagation of the pulse through them. The dispersion-tuned fiber laser we model is shown in Figure 1.

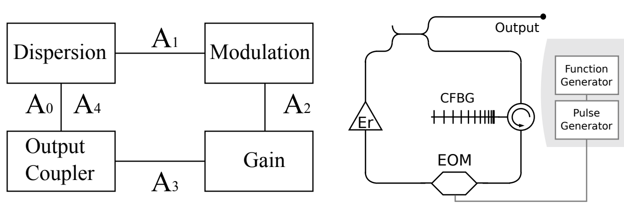


Fig. 1. Left: Schematic of the optical elements in the cavity. Right: Programmable laser schematic using fiber as a gain medium, chirped fiber Bragg gratings (CFBG) as a dispersive medium and an electrooptic modulator (EOM) for modulation.

The master equation describing the average propagation and its ansatz is given by [3]:

$$\frac{\partial A}{\partial \tau} = -\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\varepsilon}{2}T^2 A + \frac{g}{2}A \qquad A_n(T) = \sqrt{\frac{P_{cn}}{(1-iC_n)}}\exp\left[-\frac{\Omega_n^2 T^2}{2(1-iC_n)}\right]\exp(i\phi_n) \tag{1}$$

where β_2 is the net dispersion, g the net gain and ε is the net time filtering effect of the modulation over the cavity roundtrip τ on the amplitude A. No gain bandwidth terms exist in the equation since the resulting spectral width is typically much smaller than the gain bandwidth. The solution is a gaussian pulse of spectral pulsation $1/e$ half-width of Ω, of frequency chirp C and peak power P_p. The subscript used in the discrete model indicates the optical element where the pulse is considered. The discrete model uses similar equations where the propagation is done through one optical element at a time and no averaging is done over one roundtrip. The resulting pulses for both models are:

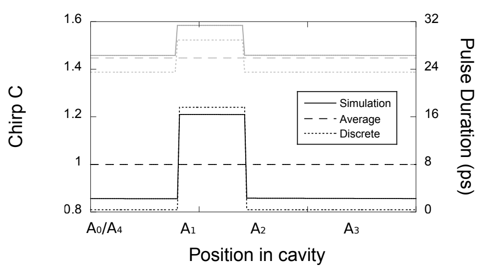
$$\begin{array}{llll} \text{Average model} & C = \mathrm{sgn}(\beta_2) = \pm 1 & \Omega^2 = \sqrt{2\frac{\varepsilon}{|\beta_2|}} & B = 1 \\ \text{Discrete model} & C_0 = \frac{\beta_2 \Omega_0^2}{2}\left(\frac{\Omega_0^2}{\varepsilon} - 1\right) & \Omega_0^4 = \frac{\varepsilon^2}{2}\left(1 + \sqrt{1 + \frac{16}{\beta_2^2 \varepsilon^2}}\right) & B = \frac{\max(\Delta T_j)}{\min(\Delta T_j)} = \frac{1}{|C_0|} \end{array} \tag{2}$$

Both models differ notably, since the chirp only depends on the cavity parameters in the discrete model. The expressions of the bandwidths are also distinct. The discrete model also shows cavity invariants that exist regardless of the position of the elements such as $C_0 C_1 = 1$. A breathing ratio B inside the laser cavity can be defined using the discrete model as the ratio of the largest pulse duration over the smallest over a roundtrip. When $C_0 = 1$, there is no breathing, which is implicitly assumed by the average model which supposes small variations. Let us define the low selectivity limit where the dispersion is much smaller than the effect of the modulator, so that a large bandwidth will go through the modulation window. Let us also consider the high selectivity limit where the dispersion is much larger than the modulation window. With $\varepsilon \propto T_M^{-2}$, the modulation window half-width at $1/e$ the limits are shown in Eqs (3). We see that the discrete model becomes the average model in the low selectivity limit where the pulse undergoes only small variations. Whereas the chirp does not depend on the cavity parameters in the low selectivity limit, it is highly dependent in the high selectivity limit and shows a large variation in the cavity since $C_0 C_1 = 1$.

$$\begin{array}{llll} \text{Low Selectivity} & |\beta_2| \ll \frac{4}{\varepsilon} & C_0 = \mathrm{sgn}(\beta_2)\left(1 - \sqrt{\frac{|\beta_2|}{2T_M^2}}\right) \approx \mathrm{sgn}(\beta_2) & \Omega_0^2 = \sqrt{\frac{2\varepsilon}{|\beta_2|}} \\ \text{High Selectivity} & |\beta_2| \gg \frac{4}{\varepsilon} & C_0 = \frac{1}{\varepsilon\beta_2} \approx 0 & \Omega_0^2 = \varepsilon\left(1 + \frac{2}{\varepsilon^2\beta_2^2}\right) \approx \varepsilon \end{array} \tag{3}$$

2. Validation and Conclusion

To validate the model, we compare the discrete model to numerical simulations and experimental data as shown in Figure 2. We also include the average model for comparison. There is an accurate agreement overall between the values of the various models and simulations and the experimental results. While the average model gives a concrete estimate, it can be seen from the left figure that it does not describe the fluctuation of the chirp and the pulse duration in the cavity. From the table, we see however that the experimental data show larger chirp and spectral width than what is predicted by the models and simulations. This broadening is explained by self-phase modulation, which was not included in the model but is still present in the laser even though it is operated near threshold. This explains why the average model yields a better fit than the simulation, since it overestimates the chirp and the spectral width.



D= -40 ps/nm T_M = 20 ps	Experiment	Simulation	Average model	Discrete model
$\Delta\lambda$ (pm)	190	178.9	192.7	193.8
Δt_0 (ps)	28.0	26.3	25.9	23.5
Chirp C_0	1.10	0.856	1	0.810
Δt_1 (ps)		31.4	25.9	28.9
Chirp C_1		1.21	1	1.24

Fig. 2. Left : Comparison between the numerical simulations and the models. Comparison between the pulse parameters obtained from the experiment, the numerical simulation, the average model and the discrete model for D = -40 ps/nm and T_M = 20 ps.

We developed a discrete model for dispersion-tuned laser which describes well to both simulations and experiments. The model highlights the variations of the pulse parameters inside the laser which are not taken into account in an average model and that proves useful to design the laser.

References

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